Fast, curvature-based prediction of rolling forces for porous media based on a series of detailed simulations

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Abstract
Using thermal spraying various surface coatings consisting of different material compositions can be manufactured. Besides different solid phases the resulting coating microstructure often contains a non-negligible amount of pores. In this context a roller burnishing process with a hydrostatic ball-point-tool is examined to compact the thermally sprayed coating, thereby reducing porosity. The rolling process is performed by a robot on free-formed workpieces. A simulation concept for the prediction of forces in a robot-guided roller burnishing process based on a series of detailed ABAQUS simulations is presented. It is shown that, based on these test configurations, the process forces can be calculated much faster and with sufficient precision. Thereby an optimal rolling path, which requires the least amount of normal force to be applied, can be determined efficiently leading to the decision whether a specific robot is equipped to handle the path. Furthermore, the described approach may be used as a pattern to apply similar methods to other engineering problems where accurate simulative solutions exist, but cannot be applied to problems of realistic size due to their expenditure of time.

Key words: rolling, simulation, force prediction, database, curvature, free-formed surface

1. Introduction and Motivation

Computer simulations are used in a variety of computer-aided engineering (CAE) applications to support the development and manufacturing process of new products [1]. The wide area of CAE includes computer-aided design applications (CAD), multi-body simulations and thermal simulations as well as simulations for fluid dynamics, robotics, fluid structure interaction and more. Also, a lot of powerful software packages, e.g., ABAQUS [2], ANSYS, CATIA, SIMULINK or SOLIDWORKS exist, which support different kinds of applications. These applications greatly reduce the time and cost during the development of new products by replacing experimental studies with virtual experiments. Nevertheless, neither the effort to create a complex simulation fitted to the needs of specific process nor the computation time are negligible, especially in fields that demand highly accurate results. In our specific case - the prediction of the process forces in a robot-guided rolling process on free-formed workpieces - the computation time of the used finite element simulation, which is set up in ABAQUS, strongly depends on the number of elements and the length of the robot motion path. To solve this problem a simulation approach, that uses detailed results from a precomputed set of simulations to predict the process forces for arbitrary inputs, has been developed.

In the literature different database approaches and applications are reported. Bae et al. [3] introduced a guideline in the tool design stage for sheet metal forming tools to reduce the cost of product development by a simulation based prediction model of the draw-bead restraining force. Jacobsen [4] presents an approach which combines a multi-level superelement concept with database concepts in an FE program. Many researchers are interested in characterizing the macroscopic constitutive behavior of heterogeneous materials. One approach to obtain this information is to

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systemically create a database of macroscopic stress-strain states which can than be employed in a structural analysis. A database methodology is presented in Temizer and Wriggers [5] in order to characterize the constitutive behavior of macroscopically orthotropic, nonlinear elastic composites. Temizer and Zohdi [6] discussed the concept of a material map, which identifies the constitutive behavior of a material in a discrete sense. A methodology by which the macroscopic nonlinear behavior of a structure made of composite materials can be analyzed by only pre-calculating the homogenized nonlinear material behavior is presented in Takano et al. [7]. A database approach has been applied to inelastic micromechanics problems by Ghosh et al. [8] to model the variation of a damage parameter. Pellegrino et al. [9] presented a database approach to create a piecewise continuous plasticity yield surface. Canal et al. [10] presented a methodology for predicting the failure behavior of a fiber-reinforced composite by generating a failure surface on basis of a RVE of the composite microstructure. In Tan and Zabaras [11] a database approach was presented for the multiscale modeling of alloy solidification. van der Bos and Geurts [12] used a database approach to quantify the total simulation error for the used material model. In Knezevic et al. [13, 14] an approach is developed on basis of a database to overcome the long computation time in crystal plasticity. A completely different goal of a database approach can be found in [15] where an active database approach is used to enable exchange of engineering information among distributed team members in a timely manner. As shown in these examples, database approaches have been used in a wide range of fields and they are mainly used to reduce the computation time.

Problem statement

In thermal spraying metallic and non-metallic surface coatings are manufactured by melting the coating materials in the form of powders or wires in an oxy-fuel gas flame, a plasma jet or an electrical arc and accelerating them towards the surface to be coated by means of the expanding combustion gases or a separate carrier gas. On the surface the impacting particles flatten, cool and solidify and thereby form a layered coating on the workpiece. Thereby thermally sprayed coatings can increase the wear resistance of the coated object, provide enhanced corrosion detection, form a thermal barrier layer or enhance other properties of the workpiece [16, 17]. An important property of a spray coating is its porosity [18], which is defined as the ratio of the volume of pores to the total volume (see figure 2). While in some applications a large porosity may be of advantage, e.g., in the manufacturing of thermal barrier coatings [19], it is unfavorable in the production of wear resistant coatings, since the pores significantly reduce the integrity of the coating structure [20]. Under load high stresses are induced at pore boundaries which facilitate the formation of cracks and may ultimately cause the failure of the structure. Especially in our field of research, the application of thermally sprayed hard materials as wear resistant coatings to enforce deep drawing tools, the coating undergoes high normal and tangential stresses in the deep drawing process [21]. In this context a roller burnishing process with hydrostatic ball-point-tools is examined to compact the thermally sprayed coating thereby reducing porosity (cp. figure 1 and 3). Furthermore, strip drawing tests showed that the resulting surface texture facilitates the storage of lubricant on the
tool surface during drawing, which in turn can promote fluid dynamic effects and enhance the tribological properties of the tool [22]. Since deep drawing tools are free-formed in general, the burnishing process has to be carried out by an industrial robot with multiple axes following a tool motion path covering the free-formed workpiece surface. To achieve a uniform indentation of the coating structure along the robot motion path, a correlation between the depth of indentation and the corresponding applied normal force has to be established. The simulative prediction of the forces along a given robot motion path, required to achieve a desired depth of indentation, is the main subject of this paper. Based on this prediction the path of the hydrostatic ball-point-tool can be chosen and two important issues can be resolved: First, the normal force that needs to be applied by the robot to reach the desired indentation can be computed at each position on the motion path and second, the maximum force on a given path can be used to estimate whether a specific robot is equipped to handle the path.

2. Simulation method

To predict the process forces a three-dimensional finite element simulation has been implemented using the commercial simulation software ABAQUS/Standard [2]. Since the simulation using ABAQUS is very time consuming, it is nearly impossible to directly apply the simulation to a full scale robot path and workpiece. Thus, the main idea is to simulate the burnishing process only on representative surface patches of small size and to use the results to build a database, that can then be used to predict the forces on arbitrary surfaces and paths.

The following section describes the creation of the database and the choice of representative configurations for the ABAQUS simulation. Section 2.2 deals with the ABAQUS simulation itself including the used material model formulation and section 2.3 gives an overview of the results and the main factors for the process forces. Finally, section 2.4 describes how the results of the ABAQUS simulations in the database can then be applied to different surfaces and paths.

2.1. Examined configurations

The database used for the prediction of process forces consists of simulation results from ABAQUS simulations of the burnishing process for different geometric configurations. As input, the ABAQUS simulation requires the geometry of the surface to be burnished, the motion path of the hydrostatic ball-point-tool on the surface and the desired indentation depth, all of which have a significant influence on the required normal forces. The result of every simulation run is a single normal force value (see section 2.2) which is saved in the database.

The considered surface patches used in the ABAQUS simulation are of the form

\[
g_{a,b}(u, v) = au^2 + bv^2 \quad \text{with} \quad a, b = \left\{ 0.025 \cdot \frac{1}{\text{min}} \cdot k \mid k = -3 \ldots 3 \right\}
\]

(1)
and $S_{a,b}(u,v) = (u,v,g_{a,b}(u,v))^T$ in three-dimensional parametric representation, respectively. The interval between $-0.075 \frac{\text{mm}}{}$ and $0.075 \frac{\text{mm}}{}$ for the coefficients $a$ and $b$ was chosen based on the radius $r = 6.5 \text{mm}$ of the used ball-point-tool and ensures that the curvature of the surface is always smaller than the curvature $\kappa = 1/r \approx 0.154 \frac{1}{\text{mm}}$ of the tool. This prevents the ball from being in a large-area contact with the surface, which would be unfeasible. In total, (1) describes elliptic and hyperbolic paraboloids with different curvatures in the principal directions induced by $a$ and $b$ (see figure 5). To be used in ABAQUS, the surfaces are extruded in $-z$ direction and subdivided into three-dimensional hexagonal finite elements as described in section 2.2.

For each of these 49 different surface configurations three different depths of indentation $d_i$ were investigated, namely 25, 50 and 100$\mu$m, resulting in theoretical indentation widths of 1.14$\text{mm}$, 1.61$\text{mm}$ and 2.27$\text{mm}$. Test simulations with a flat surface showed, that stresses decay to nearly zero at a distance of approximately 5$\text{mm}$ from the path center, so surface patch dimensions of $(u,v) \in [-5,5]^2$ were chosen. Furthermore, as shown in figure 7a, two different burnishing paths were investigated on the surface: a ‘horizontal’ path along the mid $x$-axis

$$p_h = S(u,0) + (r - d_i) \cdot \mathbf{n}_S(u,0) \quad , \quad u \in [-5,5]$$

(2)

and a diagonal path

$$p_d = S(u,v) + (r - d_i) \cdot \mathbf{n}_S(u,v) \quad , \quad u = v \in [-5,5],$$

(3)

where the indices $a, b$ of $S(u,v)$ have been dropped for better readability and

$$\mathbf{n}_S(u,v) = \frac{\frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}}{\lVert \frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v} \rVert}$$

(4)

describes the surface unit normal at $S(u,v)$. The last summand in the path formulations provides the offset $r - d_i$ of the ball-point-tool center from the surface, resulting in the desired depth of indentation $d_i$. It should be noted that switching the coefficients $a$ and $b$ in (1) is equivalent to using a vertical path $p_v = S(0,v) + (r - d_i) \cdot \mathbf{n}_S(0,v)$ instead of a horizontal one.

Altogether, a total of $49 \cdot 3 \cdot 2 = 294$ different ABAQUS simulations have been conducted to build the database.

2.2. ABAQUS simulation

All used simulation models consist of three segments: rolling ball, coating and substrate. The rolling ball-point-tool is modeled as rigid due to its high stiffness compared to coating and substrate. According to the experimental background the ball is modeled with a radius $R = 6.5 \text{mm}$. The material of the substrate is C45 and the coating consists of an aluminum-silicon alloy. Whereas the C45 substrate is modeled as bulk material the AlSi coating includes a non-negligible amount of porosity. Details about the used material model formulation are given in the next section. Since the rolling ball-point-tool is modeled as a rigid body, its displacement can be defined by the displacement of a single point, in this context the reference point. The displacement of this reference point determines the movement of the rolling ball and the total reaction force acting on the ball during the simulation can be measured at this point. Inverting this reaction force yields to the amount of force that needs to be applied by a robot handling the ball-point-tool to achieve the desired indentation depth which has been applied as a constant offset to the movement path as described in section 2.1.

The coating and substrate models used in the following were all built in the same manner. First of all, the surface is subdivided into rectangular elements. This discretization was chosen to consist of $30 \times 30$ elements for a surface geometry of $5 \times 5 \text{mm}^2$ after performing a convergence analysis. Then, the two-dimensional surface models are converted into three-dimensional volume models by performing an extrusion in two steps. The first step creates hexagonal volume elements for the AlSi coating, which is modeled with 10 equally thick layers yielding a total thickness of 500$\mu$m allowing the investigation of even small penetration depths. In the second step the extrusion is carried out using steel substrate, which is modeled with five layers yielding a total thickness of 1$\text{mm}$. Based on a convergence analysis, a thickness of 1$\text{mm}$ for the steel substrate was found to be sufficient to minimize boundary effects. Both segments - the coating and the substrate - are tied together and fixed boundary conditions are applied on the bottom layer of the steel substrate.

The evaluation of the reaction force is done for the surface segment in the middle $\in [-2.5,2.5]$ to reduce and minimize boundary effects. The resulting force of the single surface patches is computed by applying a median filter with a kernel size of about one element length which results in a straight with a constant value representing the necessary force.
2.2.1. Material model formulation

In accordance with the original local Gurson-based model formulation [23], the following model presumes local rate-independent material behavior, elastic isotropy, quasi-static loading and isothermal conditions. The formulation [24] is based on the assumption of large deformation and that the local inelastic deformation does not affect the elastic material response [25]. The hardening is assumed to be isotropic. Let $F = \nabla \chi$ be the corresponding deformation gradient $\ln V_E := \frac{1}{2} \ln (FC^{-1}F^T)$ the elastic left logarithmic stretch tensor, $C_p$ the inelastic right Cauchy-Green deformation tensor and $\alpha_p$ the accumulated inelastic deformation. In the context of the assumptions of small elastic strain appropriate for metals, the free energy is given by the additive form

$$\psi(\ln V_E, \alpha_p) = \frac{1}{2} \kappa_0 \text{tr}(\ln V_E)^2 + \mu_0 |\text{dev}(\ln V_E)|^2 + \psi_H(\alpha_p)$$ \hfill (5)

of $\psi$ into elastic and hardening contribution. The elastic part is given by the Hooke form in terms of the elastic compression moduli $\kappa_0$ and shear moduli $\mu_0$. In (5) $\text{tr}(\ln V_E)$ represents the trace, and $\text{dev}(\ln V_E)$ the deviatoric part of $\ln V_E$. For the Kirchhoff stress $K$ follows from (5) the hyperelastic relations

$$\text{tr}(K) = \text{tr}(\partial_{\ln V_E} \psi) = \partial_{\text{tr}(\ln V_E)} \psi = 3 \kappa_0 \text{tr}(\ln V_E),$$
$$\text{dev}(K) = \text{dev}(\partial_{\ln V_E} \psi) = \partial_{\text{dev}(\ln V_E)} \psi = 2 \mu_0 \text{dev}(\ln V_E),$$ \hfill (6)

and for the yield stress

$$\sigma_y = \partial_{\alpha_p} \psi.$$ \hfill (7)

In the following the material behavior is assumed to undergo linear hardening

$$\psi_H(\alpha_p) = \sigma_{y0} \alpha_p + \frac{1}{2} h_0 \alpha_p^2,$$ \hfill (8)

where $\sigma_{y0}$ represents the initial yield stress and $h_0$ the linear hardening modulus. In the thermodynamic formulation [26, 27], the evolution of the internal variables $\ln V_E$ and $\sigma_y$ are determined by the forms

$$D_p = \lambda(\partial_K \phi),$$
$$\alpha_p = -\lambda (1 - f)^{-1}(\partial_{\sigma_y} \phi)$$ \hfill (9)

for the inelastic rate of deformation $D_p$ with respect to the current configuration depending on the plastic multiplier $\lambda$, void volume fraction $f$ and the Gurson yield function, modified by Tvergaard [28] to

$$\phi(T, \sigma_y, f) = \frac{3}{2} \frac{|\text{dev}(T)|^2}{\sigma_y^2} - 1 + 2 q_1 f \cosh \left( \frac{q_2 \text{tr}(T)}{2 \sigma_y} \right) - q_3 f^2.$$ \hfill (10)

Here $T$ denotes the Cauchy stress and $q_1, q_2$ and $q_3$ are material parameters. According to the original Gurson model the parameters are chosen all equally as $q_1 = q_2 = q_3 = 1$.

The void development is assumed to occur by (strain-controlled) void nucleation and growth [29, 30] which are assumed to be uncoupled which results in the split

$$\dot{f} = \dot{f}_{\text{growth}} + \dot{f}_{\text{nucleation}} = (1 - f) \text{tr}(D_p) + A(\alpha_p) \dot{\alpha}_p$$ \hfill (11)

for the evolution of $f$ into growth and nucleation parts. $A$ describes the coefficient for strain-controlled void nucleation via

$$A(\alpha_p) = \frac{f_n}{\sqrt{2\pi} s_n} \exp \left( -\frac{1}{2} \frac{(\alpha_p - \alpha_{p,n})^2}{s_n^2} \right)$$ \hfill (12)

where $f_n$ is the volume fraction of the nucleated voids, $\alpha_{p,n}$ the mean value and $s_n$ the standard deviation of the necessary nucleation strain. However, nucleation only starts in tension loading. Therefore the term $\dot{f}_{\text{nucleation}}$ is negligible in case of rolling, where compressive loading conditions occur inside the coating.

With this material model formulation the compaction process of the pores inside the coating can be idealized described.
For simplicity the AlSi coating as well as the steel substrate are modeled as elastic-plastic with linear hardening. The AlSi coating is modeled with the presented material model whereas the steel substrate is modeled without porosity contribution. The chosen material parameters are displayed in Table 1. Here it is assumed that the porosity is homogeneously distributed inside the AlSi coating with a void volume fraction of 20%.

### 2.3. Preliminary findings

In this section results regarding the single surface patches are given. On the single surface patches the results are quite homogeneous (see figure 4a) except of the inlet and outlet region. Therefore only the results of a part of the section around the middle axis are considered for the database as described in section 2.2. Excluding this inlet region a uniform compaction along the rolling path is observable. Due to the amount of porosity compared to the penetration depth no spill over of the material at the border of the contact area between tool and part can be observed. A compaction occurs for all surface patches only directly below the contact zone due to the lower curvature of the part compared to the tool. Such results can be observed in figure 4b) which shows the distribution of the porosity in a cut-view of a planar surface patch.

Figure 5 shows the necessary compaction force for an indentation depth of 100μm. At the characteristic points the corresponding surface patches are shown. As seen the necessary compaction force depends non-linearly on the principal curvature values. The resulting force is not symmetric to $\kappa_1$ and $\kappa_2$ but rather depends on the rolling direction. The value of the force is increasing from concave to convex surface patches due to the fact that more material has to be displaced in the case of convex surfaces. Especially for values for the principal curvature higher than 0.1 m⁻¹ the force is increasing disproportionately high. The reason for this lies in the curvature of the rolling ball itself. The curvature of the ball is $\kappa = \frac{1}{R} = 0.1538\frac{1}{\text{mm}}$ which is relatively close to the maximum principle curvature value 0.15 m⁻¹.

In figure 6 the necessary forces are plotted against the penetration depth for different curvature combinations. It can be observed that the force depends nearly linearly on the penetration depth for all investigated combinations. This behavior was also observed for diagonal rolling paths. Additionally it also indicates the non-linearity between the force and principle curvature values. For the diagonal rolling paths it can be observed that the force curve for $\kappa_1 = 0.15$, $\kappa_2 = -0.15$ coincides with the curve for $\kappa_1 = -0.15$, $\kappa_2 = 0.15$ due to the symmetry.

<table>
<thead>
<tr>
<th>material</th>
<th>$E$ [MPa]</th>
<th>$\nu$ [-]</th>
<th>$\sigma_y$ [MPa]</th>
<th>$\sigma_{y,10%}$ [MPa]</th>
<th>porosity [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel</td>
<td>210000</td>
<td>0.3</td>
<td>300</td>
<td>1000</td>
<td>-</td>
</tr>
<tr>
<td>AlSi</td>
<td>70000</td>
<td>0.33</td>
<td>65</td>
<td>150</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1: Material parameters
2.4. Force prediction

To predict the normal force in the burnishing process, a simulation tool has been developed using the software package MATLAB [31]. As input, the simulation uses an arbitrary three-dimensional workpiece surface

\[ f(u, v) = (x(u, v), y(u, v), z(u, v)) \in \mathbb{R}^3 \]

in parametric representation and a \( \delta t \) time-discrete tool path

\[ p_i = \left( u(i \cdot \delta t), v(i \cdot \delta t) \right)^T \]

in the definition domain of the surface. Furthermore the desired indentation depth, 25, 50 or 100\( \mu \)m, has to be specified.

As can be seen from the preliminary findings (section 2.3), the magnitude of the normal force acting on the tool does not only depend on the magnitude of the curvature but also on its sign and the relative orientation between rolling direction and the principal curvature directions. Thus, in our simulation approach, the process force at each path point \( p_i \) is calculated on the basis of these parameters. Assuming the input path is sufficiently smooth, the 3d-rolling direction can simply be approximated by

\[ \mathbf{d}_i = p_{i+1} - p_i. \]

For the described configurations in section 2.1 contained in the lookup database, the rolling directions are simple \( (x, 0) \) for the horizontal paths and \( (x, y) \) for the diagonal ones. The principal curvatures \( \kappa_{\text{max}} \) and \( \kappa_{\text{min}} \) of the surface and their respective principal directions \( c_{\text{max}} \) and \( c_{\text{min}} \) are determined by the eigenvalues and eigenvectors of the
In the database. Thus, in total, a trilinear interpolation between eight test configurations in the database is performed based on the two principal curvatures and the tool movement direction, as exemplarily shown in figure 8.

\[ \nabla \mathbf{p}_i = \text{IFF}_{\mathbf{p}_i}^{-1} \cdot \text{IFF}_{\mathbf{p}_i}, \]

where

\[ \text{IFF}_{\mathbf{p}_i} = \begin{bmatrix} \|f_u(p_i)\|^2 & \langle f_u(p_i), f_v(p_i) \rangle \\ \langle f_u(p_i), f_v(p_i) \rangle & \|f_v(p_i)\|^2 \end{bmatrix} \]

and

\[ \text{IFF}_{\mathbf{p}_i} = \begin{bmatrix} \langle n_f(p_i), f_{uu}(p_i) \rangle & \langle n_f(p_i), f_{uv}(p_i) \rangle \\ \langle n_f(p_i), f_{uv}(p_i) \rangle & \langle n_f(p_i), f_{vv}(p_i) \rangle \end{bmatrix} \]

represent the first and second fundamental form of the surface at \( \mathbf{p}_i \). In the above equations, \( f_u, f_v, f_{uu}, f_{uv} \) and \( f_{vv} \) denote partial derivatives in the respective directions, \( n_f(p_i) \) is the surface unit normal according to (4) and \( \langle \cdot, \cdot \rangle \) denotes the dot product.

The current tool movement direction \( \mathbf{d}_i \) can now be related to the principal directions using the angle \( \alpha \) between \( \mathbf{d}_i \) and, e.g., \( c_{\text{max}} \) in the tangent space \( T_{\mathbf{p}_i} \mathbf{s} \) as shown in figure 7. Since \( c_{\text{min}} \perp c_{\text{max}} \), the angle between \( \mathbf{d}_i \) and, e.g., \( c_{\text{min}} \) need not to be considered. Furthermore it can be seen in figure 7b that, at every point \( \mathbf{p}_i \), there are up to four tool movement directions (green arrows) with identical angles \( \angle(\mathbf{d}_i, c_{\text{min}}) \) and \( \angle(\mathbf{d}_i, c_{\text{max}}) \) and thus, with identical curvature. This is the case, because \( c_{\text{min}} \) and \( c_{\text{max}} \) are undirected in the sense that \( -c_{\text{min}} \) and \( -c_{\text{max}} \) also represent the principal curvature directions. Due to this symmetry only tool movement directions in the first quadrant of the tangent space are considered in the following.

For the test configurations in the database, the principal curvature directions coincide with the \( x \)- and \( y \)-axes of the test surface. Thus, to match a test configuration to the curvatures at a point \( \mathbf{p}_i \), the \( x \)- and \( y \)-axes point in the \( c_{\text{max}} \) and \( c_{\text{min}} \) directions or vice versa - depending on the values of the coefficients of \( a \) and \( b \) in (1). This implies that the rolling directions of the test configurations, which are shown as red arrows in figure 7b, also point in the direction of \( c_{\text{max}}, c_{\text{min}} \) or in the diagonal direction \( c_{\text{min}} + c_{\text{max}} \) (A, C and B in figure 7b). If the tool movement direction \( \mathbf{d}_i \) would exactly match one of these directions A, B or C, the relative orientation of \( \mathbf{d}_i \) and the principal directions at \( \mathbf{p}_i \) would be the same as in the respective test configuration. In general, when \( \mathbf{d}_i \) lies in between two of the rolling directions A, B and C, the wanted force value is linearly interpolated from test configurations with neighboring directions - in the example depicted in figure 7b, this would be the A and B directions. However, since the curvatures \( \kappa_{\text{min}} \) and \( \kappa_{\text{max}} \) at \( \mathbf{p}_i \) are most likely different from the ones in the test configurations, which are determined by the coefficients \( a \) and \( b \) in (1), the exact force values to be used for the interpolation at A and B are still unknown. Based on the curvatures \( \kappa_{\text{min}} \) and \( \kappa_{\text{max}} \) at \( \mathbf{p}_i \), they can be approximated by bilinear interpolation from the discrete number of available curvatures in the database. Thus, in total, a trilinear interpolation between eight test configurations in the database is performed based on the two principal curvatures and the tool movement direction, as exemplarily shown in figure 8.

![Figure 6: Resulting force for different curvature combination over different penetration depth for a horizontal rolling direction.](image)

\[ \text{force}[\text{N}] = \text{IFF}_{\mathbf{p}_i}^{-1} \cdot \text{IFF}_{\mathbf{p}_i}, \]

where

\[ \text{IFF}_{\mathbf{p}_i} = \begin{bmatrix} \|f_u(p_i)\|^2 & \langle f_u(p_i), f_v(p_i) \rangle \\ \langle f_u(p_i), f_v(p_i) \rangle & \|f_v(p_i)\|^2 \end{bmatrix} \]

and

\[ \text{IFF}_{\mathbf{p}_i} = \begin{bmatrix} \langle n_f(p_i), f_{uu}(p_i) \rangle & \langle n_f(p_i), f_{uv}(p_i) \rangle \\ \langle n_f(p_i), f_{uv}(p_i) \rangle & \langle n_f(p_i), f_{vv}(p_i) \rangle \end{bmatrix} \]
Figure 7: a) Visualization of an exemplary test configuration including the horizontal (blue) and diagonal (red) tool paths \( p_h \) and \( p_d \) according to (2) and (3). For \((x, y) = (0, 0)\) the surface normal vector \( n_y \) and the according tangent space with the principal directions and the rolling direction are shown. b) Depiction of vectors in the tangent space \( T_{p_f} \). The green vectors are four equivalent rolling directions with angle \( \alpha \) to the principal direction. All configurations are axially symmetric with respect to the principal directions at \( p_f \).

3. Evaluation

To evaluate the force prediction method simulations, their results based on the created database were compared to the results of ABAQUS-based simulations for a complex workpiece surface and different tool movement paths. Since the results with different paths all showed the same deviations and characteristics between the predicted and the accurately simulated ABAQUS-solutions, only one exemplary simulation is discussed in the following.

Figure 9 shows the workpiece, which was modeled with the aim to provide a smooth surface with varying curvature values limited approximately by the curvature of the hydrostatic ball-point-tool. The tool movement path was chosen as a circular path with a diameter of approximately \( d_v = 2r_p \approx 40 - 45 \text{mm} \). Figure 10 shows the development of the minimum and maximum curvatures and the curvature in the direction of the rolling path against the simulation time ranging from 0 to 14 seconds.

For the ABAQUS simulation, two different meshing strategies were employed. First, a regular subdivision of the complete, 65.4\( \text{mm}^2 \) large surface in the x- and y-directions has been applied, where 120 \( \times \) 120 elements were used for the upper coating element layers and 70 \( \times \) 70 elements per layer for the steel substrate. Also, to reduce computation time for this large scale experiment, the number of layers for the coating had to be reduced to 5 in contrast to the 10 layers for the database configurations (leaving the total coating thickness unchanged). In total, this results in 96500 elements. Figure 12a shows a snapshot from the simulation.

To decrease the simulation time for the ABAQUS simulation, a second, rotationally symmetric meshing approach has been employed. Therein the subdivision of each layer was performed in a two-dimensional polar coordinate system \((r, \phi)\), whose point of origin is located at the center of the circular tool movement path. The finite elements were created by subdividing \( \phi \in [0, 2\pi] \) into 120 segments (along the tool path) and subdividing \( r \in [r_p - 5\text{mm}, r_p + 5\text{mm}] \) into 40 segments. This means only a 10mm wide strip around the tool path with radius \( r_p \) is considered as shown in figure 12b. Compared to the regular subdivision the second approach reduces the number of elements approximately by half. It should be noted, that this boost of the simulation time comes at the cost of higher oscillations in the reaction forces since the element size is larger. However, by applying a median filter with a kernel size of about one element length, most oscillations could be eliminated and a good agreement between the forces for both meshing strategies could be achieved.
Figure 8: The predicted force value is calculated by trilinear interpolation of force values from the test configurations depending on the angle between the tool movement direction and $c_{\text{max}}$ as well as the principal curvature magnitudes.

Figure 9: The sample surface used in the evaluation of the force prediction method. The red circle marks the path of ball-point-tool on the surface, i.e., where the tool is in contact with the workpiece. The coarse grid on the surface is only shown for a better recognizability, it does not represent a used subdivision (which is much finer).
Figure 10: Developing of the minimum and maximum curvature in the principal directions and the curvature in the rolling direction (red) along the path. Naturally the curvature in the rolling direction is bounded by the minimum and maximum curvature.

Figure 11 shows the predicted forces and the forces calculated by ABAQUS are plotted against the simulation time. It can be seen, that the predicted forces are in good agreement with the ABAQUS simulation result. They disagree only at the first and last second of the simulation time, which is to be expected because the ball-point-tool is slowly lowered onto its contact position with the surface at the beginning and raised from the surface at the end. The average deviation of the prediction measured over the simulation time from one to 13 seconds is \( \approx 14.5 \text{N} \) or \( \approx 2.45\% \) and a maximum deviation of 5.46\%. For the comparison of the simulation performance both simulations were conducted on an Intel Core2Duo P8600 machine with 2.4GHz and 4GB memory running Ubuntu Linux 8.04. While the complete simulation processes with ABAQUS take about 400 hours for the regularly subdivided workpiece with 100,000 elements and 142 hours for the rotationally symmetric meshing approach, the MATLAB simulation only takes 48.8 seconds. This computation time includes the loading process of the database, the creation of the surface, the calculation of derivatives and principal directions along the tool path and the lookup and interpolation of values from the database. Thus, the MATLAB prediction is more than 10,000 times faster than the faster of the two ABAQUS simulations.

4. Conclusion

A simulation concept for the prediction of forces in a roller burnishing process based on a series of basic ABAQUS simulations has been presented. It has been shown that, based on these basic test configurations, the process forces can be calculated with sufficient precision and about 10,000 times faster. This speed-up makes it possible to simulate the roller burnishing process for tool paths with more realistic lengths rather than the \( \approx 125 - 140 \text{mm} \) in the test above. Also, the efficiency of the presented method allows for application in a pathless way by simply calculating the process forces at all points on the surface for a set of constant rolling directions as shown exemplary in figure 13. With this approach an optimal rolling direction, which requires the least force, can be found for every point and used during path planning.

On a more general level, the presented simulation method may be applied to other engineering problems where accurate simulative solutions exist, but cannot be applied to real problems due to their expenditure of time. Furthermore
The presented simulation approach points out that, while computer simulations are already important today to reduce costs and development time in many areas, there is still the need for even faster simulation approaches in cases where the problem is too complex to be accurately solved in an acceptable time frame.

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References

Figure 13: Visualization of the point-wise difference between the predicted rolling forces for rolling in the y-direction and in the x-direction. In the reddish areas rolling in the y-direction is preferable due to smaller forces necessary to achieve the desired indentation depth. Accordingly, in the bluish areas the x-direction is preferable. White areas cannot be rolled at all using the tool with 6.5mm radius due to a too high curvature.


